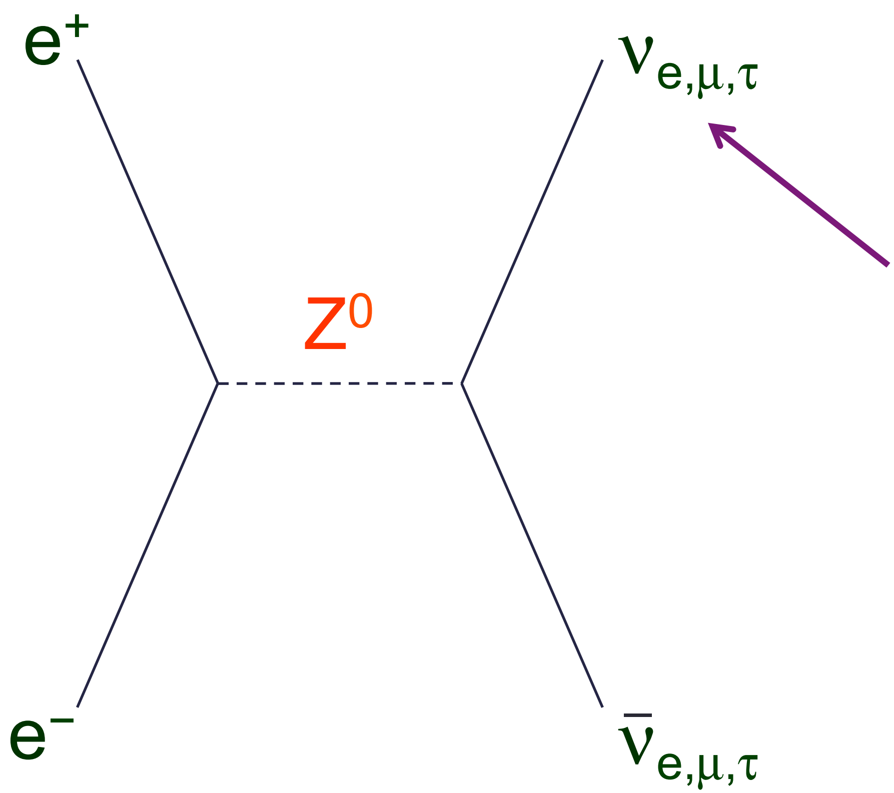
Previously, on Neutrino Physics…

- started with the well-known context of neutrino oscillations established by Super-K and SNO and other experiments
- simple and standard mathematical framework for 2-neutrino oscillations
  - and what a typical vacuum oscillation experiment can do for us
- what does it all mean?
  - Schrödinger's Cat analogy
  - Young's Two-Slit Experiment analogy
  - cast into the parlance of quark mixing
- full 3×3 PMNS mixing matrix mathematical framework
- angles, phases, what are the possible values
  - octant degeneracy and mass hierarchy
- all of the above for *vacuum* oscillations

Neutrino Production by NC

- e.g. supernova neutrinos, thermal production



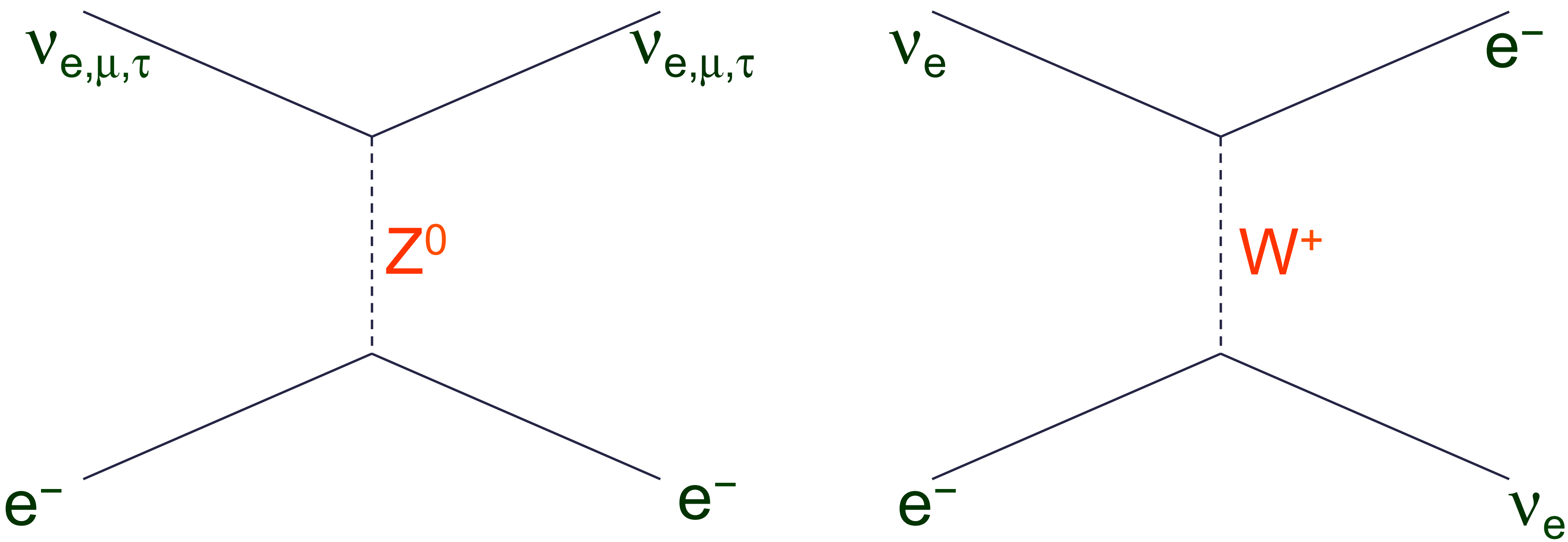
do these neutrinos oscillate?

thought provoking opener…not really related to today's material

(Wolfenstein, 1978; Mikheyev & Smirnov, 1985)

Matter-Enhanced ν Oscillations

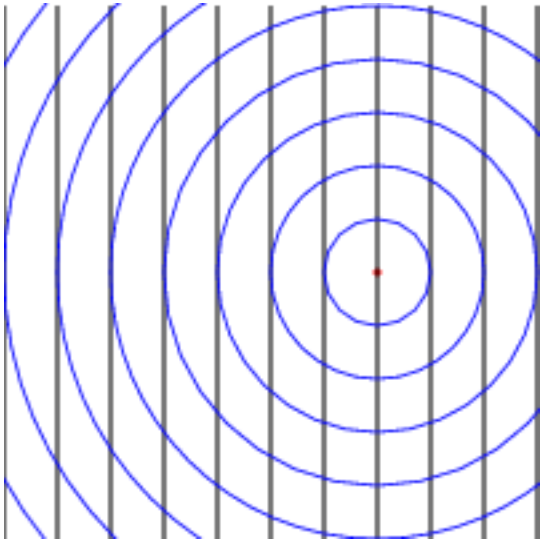
- propagation through matter affects ν_e and ν_μ, ν_τ differently [Mikheyev, Smirnov and Wolfenstein – MSW effect]
- forward-scattering amplitudes are different
- optical theorem → like an index of refraction



ν_e wavefunction phase *is affected by propagating through ordinary (dense) matter*

Plane Wave Scattering – Optical Theorem

- total cross section: $\sigma_{tot} = \frac{4\pi}{p} \mathrm{Im}[f(p,0)]$
- phase shift: $\Delta\phi(x) = \frac{2\pi}{p} N x \mathrm{Re}[f(p,0)]$



where f(p,0) is the forward-scattering amplitude

in optics, complex index of refraction:

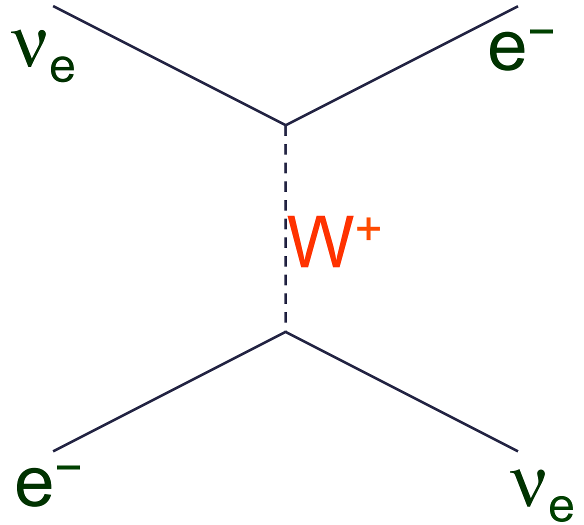
$$n = 1 + \frac{2\pi}{k^2} N f(k,0)$$

CC Forward-Scattering Amplitude

- only the additional CC interaction for ν_e is important
- the NC interaction introduces the same phase shift for all flavours and can be ignored
- forward-scattering amplitude for this diagram can be calculated

$$\mathrm{Re}[f(p,0)] = \frac{-\sqrt{2}\,G_F\,p}{2\pi}$$

$$\Delta\phi(x) = -\sqrt{2}\,G_F\,N_e\,x$$



another common approach is to translate this interaction into a potential term in the Hamiltonian: $V_{CC} = \sqrt{2}\,G_F\,N_e$

Propagating in Matter versus Vacuum

$$\left|\nu_k(t)\right\rangle = e^{-iE_k t}\left|\nu_k\right\rangle \text{ in vacuum becomes}$$

$$\left|\nu_k(t)\right\rangle = e^{-iE_k t}\sum_{\alpha\neq e} U_{\alpha k}\left|\nu_\alpha\right\rangle + e^{-i(E_k t+\sqrt{2}G_F N_e x)}U_{ek}\left|\nu_e\right\rangle \text{ in matter}$$

Hamiltonian operator is: $i\dfrac{d}{dt}$ and thus propagating through matter

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = U^\dagger\begin{pmatrix} m_1^2/2E & 0\\ 0 & m_2^2/2E\end{pmatrix}U\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} + \begin{pmatrix}\sqrt{2}G_F N_e & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

after eliminating common phase ‘E’ between ν_1 and ν_2

"Hamiltonian" for Propagating through Matter

- this describes the time evolution of flavour states (simplified 2-flavour description)

$$M^2 = U^\dagger \begin{pmatrix} m_1^2/2E & 0 \\ 0 & m_2^2/2E \end{pmatrix} U + \begin{pmatrix} \sqrt{2}G_F N_e & 0 \\ 0 & 0 \end{pmatrix}$$

- this matrix is not diagonal
- you diagonalize a matrix by finding a transformation R that rotates the non-diagonal matrix into a new basis
- the new diagonal entries are the eigenvalues
- the transformation R is the "rotation" from the non-diagonal basis vectors to the new basis of eigenvectors

Diagonalize the Matter Hamiltonian

$R^\dagger M^2 R$ where

$$\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = R \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} \cos\theta_m & -\sin\theta_m \\ \sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

solve for θ_m in terms of θ, Δm^2, E, G_F, N_e

define $A = 2E\sqrt{2}\,G_F N_e$

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{-A + \Delta m^2 \cos 2\theta} \quad \text{or}$$

$$\sin^2 2\theta_m = \frac{(\Delta m^2 \sin 2\theta)^2}{(A - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2}$$

Symmetrize and Diagonalize

$$M^2 = U^\dagger \begin{pmatrix} m_1^2/2E & 0 \\ 0 & m_2^2/2E \end{pmatrix} U + \begin{pmatrix} \sqrt{2}G_F N_e & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \frac{1}{4E}\left[(\Sigma + A) I + \begin{pmatrix} A - \Delta m^2 C_{2\theta} & \Delta m^2 S_{2\theta} \\ \Delta m^2 S_{2\theta} & -A + \Delta m^2 C_{2\theta} \end{pmatrix} \right]$$

$$A = 2E\sqrt{2}G_F N_e$$

$$\Sigma = m_1^2 + m_2^2$$

$$\Delta m^2 = m_2^2 - m_1^2$$

$$C_{2\theta} = \cos 2\theta; \quad S_{2\theta} = \sin 2\theta$$

identity matrix

units of mass squared

Eigenvalues of M^2 in Matter

$$\begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

- use formula for diagonalizing 2×2 real symmetric matrix
- effective masses squared of ν_{1m} and ν_{2m}

$$\lambda_{2,1} = \left[(\Sigma + A) \pm \sqrt{(A - \Delta m^2 C_{2\theta})^2 + (\Delta m^2 S_{2\theta})^2} \right] / 2$$

Limits and Resonance

- $A \to 0$, masses and mixing angles revert to vacuum values
- $A \gg \Delta m^2$, then $\theta_m = \pi/2$, ν_{1m} is all ν_μ
  - sort of interesting…but no oscillations (it's all $\nu_{1m} = \nu_\mu$)
- resonance condition: $A = \Delta m^2 \cos 2\theta$, then $\theta_m = \pi/4$, no matter how small the vacuum mixing angle θ
  - maximal mixing is generated at resonance

$$\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} \cos\theta_m & -\sin\theta_m \\ \sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{-A + \Delta m^2 \cos 2\theta} \quad \text{or}$$

$$\sin^2 2\theta_m = \frac{(\Delta m^2 \sin 2\theta)^2}{(A - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2}$$

Characteristic Length (again)

- for vacuum oscillations and for matter effects

$$L_V = \frac{4\pi E}{\Delta m^2}$$

$$L_e = \frac{2\pi}{\sqrt{2} G_F N_e}$$

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{-A + \Delta m^2 \cos 2\theta} \quad \text{or}$$

$$\tan 2\theta_m = \frac{\tan 2\theta}{-\frac{A}{\Delta m^2}\sec 2\theta + 1}$$

$$A = 2E\sqrt{2}\, G_F\, N_e$$

$$\tan 2\theta_m = \frac{\tan 2\theta}{-\frac{A}{\Delta m^2 \cos 2\theta} + 1} = \frac{\tan 2\theta}{-\frac{L_V}{L_e \cos 2\theta} + 1}$$

recall the discussion about the 1st and 2nd octant in the previous lecture…
2nd octant is like flipping the mass hierarchy (and a relative phase between the mass states, irrelevant) and mapping back to the 1st octant

but for matter effects, it matters!

To Resonance or not to Resonance

- if the vacuum mixing angle is in the first octant, the resonance condition is possible
- if the vacuum mixing angle is in the second octant, equivalent to flipping the mass hierarchy, the resonance condition is not possible
- you can see the 2nd octant, flipped hierarchy equivalency directly

- it is possible (not always) for an experiment to observe the matter effect and conclude which hierarchy is involved
  - the 1st and 2nd octant degeneracy can be broken

this happened for solar neutrinos (SNO)

$$\tan 2\theta_m = \frac{\tan 2\theta}{-\frac{A}{\Delta m^2 \cos 2\theta}+1} = \frac{\tan 2\theta}{-\frac{L_V}{L_e \cos 2\theta}+1}$$

To Resonance or not to Resonance

- if the vacuum mixing angle is in the first octant, the resonance condition is possible
- if the vacuum mixing angle is in the second octant, equivalent to flipping the mass hierarchy, the resonance condition is not possible
- you can see the 2nd octant, flipped hierarchy equivalency directly

- it is possible (not always) for an experiment to observe the matter effect and conclude which hierarchy is involved
  - the 1st and 2nd octant degeneracy can be broken

this happened for solar neutrinos (SNO)

$$\tan 2\theta_m = \frac{\tan 2\theta}{-\frac{A}{\Delta m^2 \cos 2\theta}+1} = \frac{\tan 2\theta}{-\frac{L_V}{\text{[illegible]}}+1}$$

For Solar Neutrino Oscillations

- the 2^{nd} octant was briefly referred to as the "dark side" (maybe still called that by old school ν, like me)
- b/c oscillations are always $\sin^2 2\theta$ and people had not looked at solutions in the second octant for solar neutrinos for a period of time (though initially they had)

The Dark Side of the Solar Neutrino Parameter Space*

André de Gouvêa
CERN - Theory Division, CH-1211 Geneva 23, Switzerland

Alexander Friedland and Hitoshi Murayama
Department of Physics, University of California, Berkeley, CA 94720, USA
Theory Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
(May 25, 2006)

Results of neutrino oscillation experiments have always been presented on the $(\sin^2 2\theta, \Delta m^2)$ parameter space for the case of two-flavor oscillations. We point out, however, that this parameterization misses the half of the parameter space $\frac{\pi}{4} < \theta \leq \frac{\pi}{2}$ ("the dark side"), which is physically inequivalent to the region $0 \leq \theta \leq \frac{\pi}{4}$ ("the light side") in the presence of matter effects. The MSW solutions to the solar neutrino problem can extend to the dark side, especially if we take the conservative attitude to allow higher confidence levels, ignore some of the experimental results in the fits, or relax theoretical predictions. Furthermore, even the so-called "vacuum oscillation" solution distinguishes the dark and the light sides. We urge experimental collaborations to present their results on the entire parameter space.

Matter Effects for Antineutrinos

- V_{CC} changes sign for antineutrinos

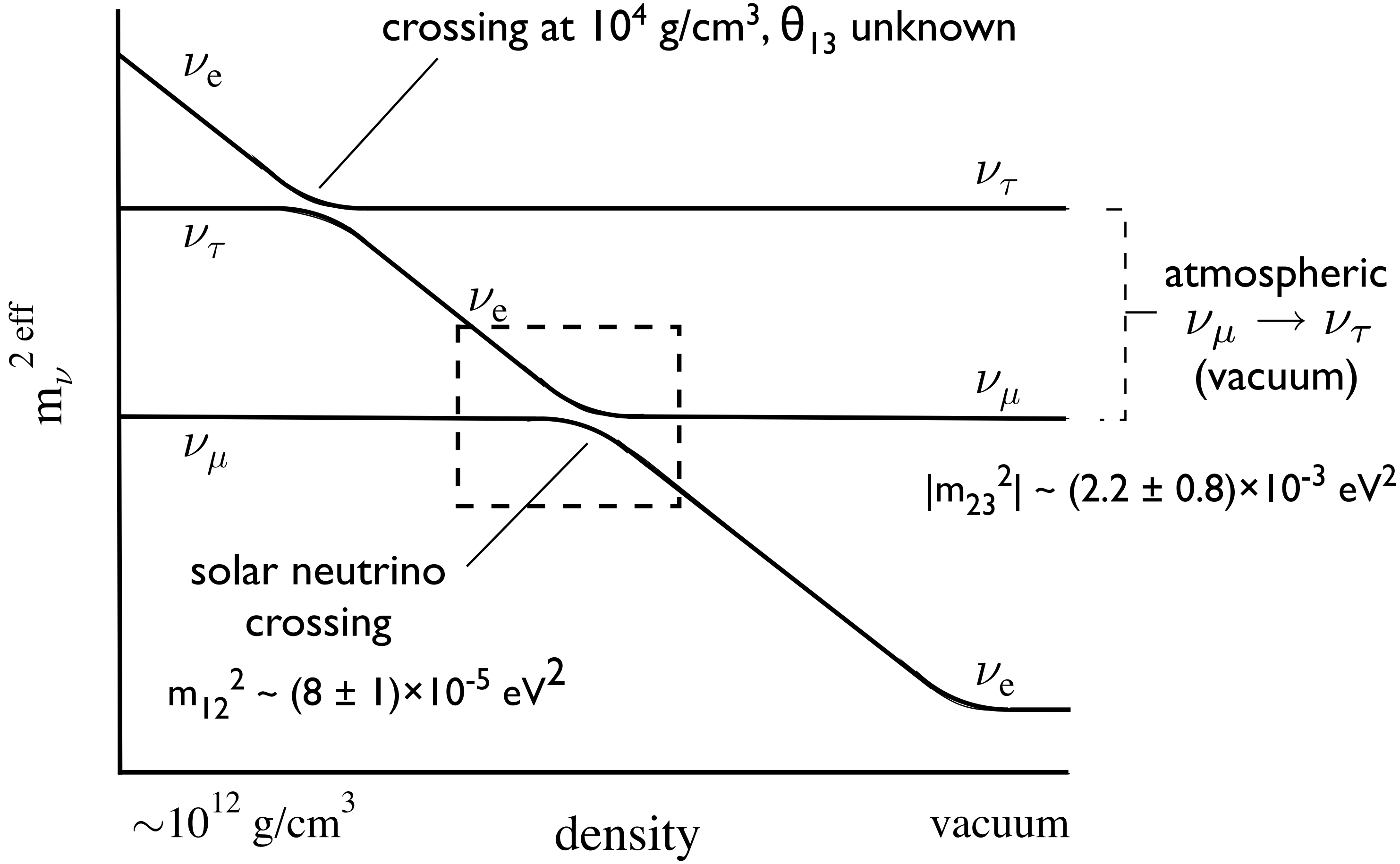
$$\sqrt{2}G_F N_e \rightarrow -\sqrt{2}G_F N_e \text{ for } \bar{\nu}_e$$

- everything said about matter effects, resonance, mass hierarchy, is reversed for antineutrinos

$$\lambda_{2,1} = \left[(\Sigma + A) \pm \sqrt{(A - \Delta m^2 C_{2\theta})^2 + (\Delta m^2 S_{2\theta})^2} \right] / 2$$

Matter Effect Versus Density

- from W.C. Haxton, arXiv:0710.2295



How to Think About Neutrinos Propagating Through Matter

- especially through matter with varying density

- sequential slab calculations
- phase matching at boundaries
- finite time difference analysis – take a step using the Hamiltonian in the matter mass basis

- at any point, can project onto flavour basis using the diagonalizing R transformation in 3×3 to get the flavour detection probability