
NB: These problems only ask for order of magnitude estimates, but for Q2 you may need to look up a few astrophysical and geophysical parameters (which is part of the exercise).

1. [**Lecture 1**] The thermal relic scenario for dark matter production in the early universe was discussed in the lectures. Cold relics require an annihilation rate $\langle \sigma v \rangle \sim 1\,\mathrm{pb}$ (in $c = 1$ units), which appears natural for a particle with weak-scale mass and couplings. Consider in more detail the full dark matter mass range that can achieve the observed relic abundance through thermal freeze-out.

   (a) For dark matter candidates that annihilate through tree-level weak interactions with $\langle \sigma v \rangle \sim G_F^2 m_{\mathrm{DM}}^2$, the requirement that $\langle \sigma v \rangle \gtrsim 1\,\mathrm{pb}$ implies $m_{\mathrm{DM}} \gtrsim \mathcal{O}(\text{few GeV})$, known as the Lee-Weinberg bound. Show that, independent of the annihilation channel, the freeze-out condition discussed in the lectures for a *cold* thermal relic only implies the more general lower bound $m_{\mathrm{DM}} \gtrsim \mathcal{O}(0.1)\,\mathrm{eV}$.

   (b) Make an estimate of the upper mass bound for a cold thermal relic, again without making assumptions about the interactions which mediate annihilation, but imposing the s-wave unitarity limit on the annihilation cross section, $\sigma \leq \pi / p_{\mathrm{c.m.}}^2$.

   (c) These constraints allow for quite a wide mass range, but can you suggest loopholes in both arguments that would expand the range further? As one variant, consider the thermal freeze-out abundance of protons. Estimate of the $p\bar{p}$ annihilation rate, and show that it is far too large to explain the observed baryon abundance, roughly one fifth of the dark matter abundance. [*NB: In this case, the observed abundance apparently arises through a mechanism (baryogenesis) which generates an asymmetry between the protons and antiproton abundance, with the proton excess surviving freeze-out. There are analogous models of 'asymmetric' dark matter.*]

2. [**Lecture 2**] A possible indirect signature of dark matter, mentioned only briefly in the lectures, is due to capture and annihilation in stars, e.g. the Sun, producing a high energy source of neutrinos that could be observed in IceCube for example. Similar arguments can be applied to the Earth, and this question considers instead the impact of DM heating.

   (a) Assuming one scatter is sufficient for capture in the Earth, make an estimate of the capture probability using a spin-independent per-nucleon scattering cross section $\sigma_N \sim 10^{-41}\,\mathrm{cm}^2$ that is still (almost) allowed for light dark matter with $m_{\mathrm{DM}} \sim$ few GeV.

   (b) Ignoring the (albeit important) details of the equilibration of capture, annihilation and evaporation rates, use the capture probability from (a) and simply assume that all the captured dark matter annihilates and has its mass converted to heat. Hence estimate the heating rate given $\rho_{\mathrm{DM}} \sim 0.3\,\mathrm{GeV/cm}^3$ locally. In comparison to the measured heat transfer from the Earth's core, is this rate a significant source of 'global warming'?

   (c) Repeat the calculation above for a white dwarf (WD) star in a denser region, e.g. where $\rho_{\mathrm{DM}} \sim 10^2\,\mathrm{GeV/cm}^3$. Determine whether the heating rate in this case is negligible or not in comparison to the typical observed WD luminosity.