TRISEP Problems
(Ann Nelson's lectures)

[1] The strong CP problem is the problem of why the neutron (and other hadrons) has such a small electric dipole moment (consistent with zero to excellent precision). This requires that the QCD parameter $\overline{\theta}$ be very close to zero (less than of order 10^{-10}). Explain why only particles with spin can have electric dipole moments (EDM)s. Also, explain why an EDM violates P and T.

[2] The Axion is a solution to the strong CP problem, which makes the QCD $\overline{\theta}$ parameter a dynamical field, $\theta\ a/f_a$, where a=$a(\vec{x},t)$. The energy density as a function of $\overline{\theta}$ may be estimated to be of order $m_\pi^2 f_\pi^2(1-\cos\overline{\theta})$, so is minimized at the CP conserving value $\overline{\theta}=0$. The axion mass, and all the axion couplings, are therefore inversely proportional to f_a. **Using dimensional analysis, estimate** the axion mass and the axion lifetime, for $f_a = 10^{11}$ GeV.

[3] The linearized equation of motion for the axion field in the early universe is

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0,$$

where H is the Hubble constant. (The effective axion mass is temperature dependent, but ignore that complication in the following). **Roughly Estimate** the temperature of the universe when the axion field begins to oscillate, for $f_a = 10^{11}$ GeV.

[4] Assuming the energy density in axions was initially of order $m_\pi^2 f_\pi^2$, and using the fact that the temperature of the universe scales as $1/R$, where R is the scale factor, and neglecting the temperature dependence of the axion mass, roughly estimate the energy density in axions today as a function of f_a.

[5] Assuming axions are all of the dark matter, how does the number density of axions scale with f_a? How does the interaction rate per axion scale with f_a?