TRISEP Neutrino Physics Homework #1

1. Pion decay at rest: $\pi^+ \rightarrow \mu^+ + \nu_\mu$

   The charged-current reaction produces neutrinos in a pure flavour state. The neutrinos emitted could have different masses m_k as there are 3 possible mass eigenstates involved. The kinematics of this two-body decay are fixed. If you know precisely $(E_\mu, \vec{p}_\mu)$ you can know precisely $(E_k, \vec{p}_k)$ of the emitted neutrino, and then you can tell what neutrino mass eigenstate was emitted.

   Derive the exact expression to solve for m_k if you know exactly $\vec{p}_\mu$. Your expression can have m_π, m_μ in it, since both are known quantities. This is just kinematics and math – no oscillations involved yet.

2. For the thought experiment above, if you measure the outgoing muon momentum precisely you can know the mass eigenstate of the emitted neutrino and you also know it is a pure flavour state, ν_μ, so how does this work consistently with quantum mechanics and neutrino oscillations?

   Does this outgoing muon neutrino oscillate? What does one observe if one places a neutrino detector at varying distances L from a pion decay-at-rest source that has this precise muon instrumentation? Does the survival probability (muon neutrino detection probability) vary with distance? If not, what's different about this muon neutrino beam (i.e. why do real muon neutrino beams produce neutrinos that oscillate)? Note that in quantum mechanics you don't actually have to make the measurement (that decoheres the quantum state) if nature makes that measurement for you.

   The actual neutrino mass splittings save us from this "paradox". It may help to use real values in your discussion, consider only two-neutrino mixing, and use $\Delta m^2 = 2.5 \times 10^{-3}$ eV^2 as the mass difference you need to be able to infer from measurement. Hint: think about measurement uncertainty for $\Delta m_k, \Delta p_\mu$ and the uncertainty principle.

3. In the lectures I showed an expression for plane waves that was manifestly Lorentz invariant. Lorentz invariance should be a consideration because neutrinos are ultra-relativistic. But, even with Lorentz-invariant plane waves, the survival probability formula was derived (with some approximations) and it has L/E in it. But, in a different reference frame, a moving observer sees $L' = L/\gamma$ (Lorentz contraction) and sees $E' = \gamma(E - vp) \approx \gamma(1 - v)E$ and there is a problem because L'/E' is not equal to L/E. Does a moving observer see different neutrino physics? Fix this problem!

4. Using the standard parameterization for three-neutrino mixing (θ_{12}, θ_{23}, θ_{13}, δ) show that this is the full expression for the survival probability.

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\theta_{13} \cos^2 \theta_{12} \sin^2(\frac{\Delta m_{31}^2 L}{4E})$$
$$-\cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2(\frac{\Delta m_{21}^2 L}{4E})$$
$$-\sin^2 2\theta_{13} \sin^2 \theta_{12} \sin^2(\frac{\Delta m_{32}^2 L}{4E})$$

Look up all the values (from PDG), get your favourite plotting software out, and plot the survival probability versus L/E with this full expression for both the normal hierarchy and the inverted hierarchy, superimposed.