TRISEP Neutrino Physics Homework #2

1. A neutrino in a pure mass eigenstate ν_2 (consider two-flavour propagation, ν_e and ν_μ) is incident upon a slab of matter of constant density. Let's use real numbers:

   $$\left|\langle \nu_e | \nu_2 \rangle\right|^2 = 0.25, \quad \left|\langle \nu_\mu | \nu_2 \rangle\right|^2 = 0.75, \quad \Delta m_{21}^2 = 8\times 10^{-5}\,\mathrm{eV}^2.$$

   The neutrino propagates 10,000 km in rock of density 3 g/cm^3. What is the probability to detect ν_e after this distance? This is, of course, Earth regeneration (day-night effect). Does the "phase" of the neutrino wavefunction at the vacuum-matter boundary affect your answer?

2. Calculate a few characteristic oscillation lengths for yourself: KamLAND reactor neutrinos at 5 MeV and solar Δm^2, T2K long baseline 1 GeV neutrinos and atmospheric Δm^2, Daya Bay reactor neutrinos at 5 MeV and atmospheric Δm^2, L_e for matter effects in the solar core with $N_e = 100\ N_{Av}\ \mathrm{cm}^{-3}$.

3. The off-axis neutrino beam kinematics are straightforward to calculate. Make the plot shown in the lecture of neutrino energy versus pion energy [GeV] for

   $$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

   with curves for the neutrino emitted in the lab frame at several different small angles from the pion boost direction. This plot shows that for a broad range of pion energies, the neutrino is emitted with the same energy helping to make a neutrino beam with an energy peak.