Electroweak Theory, SSB, and the Higgs: Lecture 1

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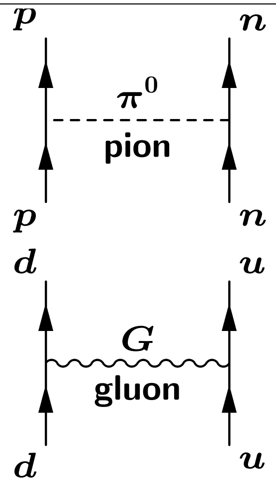
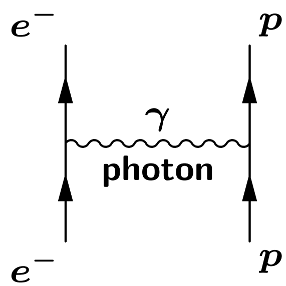
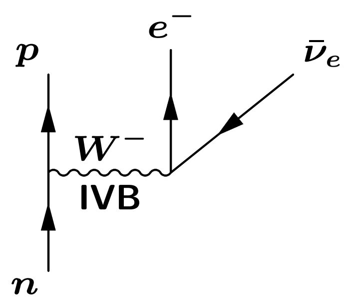
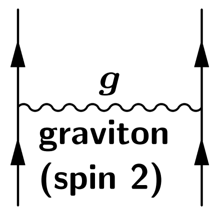
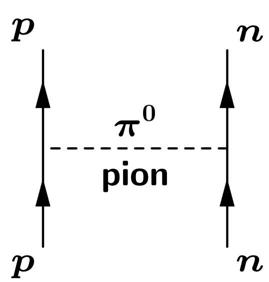
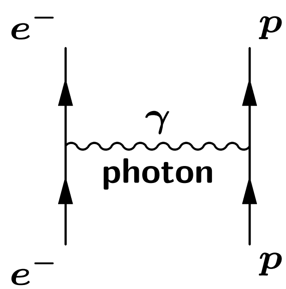
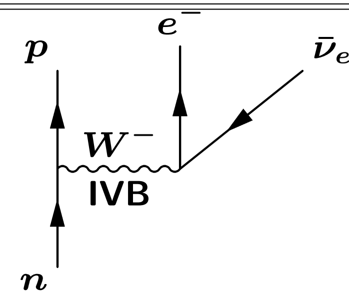
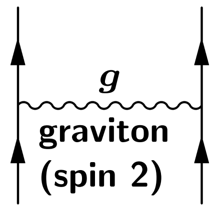
- Paul Langacker
  Institute for Advanced Study and Princeton University
  pgl@ias.edu

- References
  - PL, *The Standard Model and Beyond* (CRC Press, 2010).
    Website (contains updated section on Higgs physics):
    www.sns.ias.edu/∼pgl/SMB/
  - PL, 0901.0241 (short introduction to Standard Model)
  - J. Beringer *et al.* [pdg.lbl.gov/], *Review of Particle Physics*,
    Phys. Rev. D86, 010001 (2012)
    (numerous useful review articles)

- Topics
  1. The standard model Lagrangian
  2. The electroweak gauge interactions (after symmetry breaking)
  3. Theoretical aspects of spontaneous symmetry breaking
  4. Experimental aspects of the Higgs

- Standard model
  - $SU(3) \times SU(2) \times U(1)$ (extended to include ν masses)
    + general relativity
  - Mathematically consistent, renormalizable theory
  - Accounts for everything observed in lab to 10^{-16} cm
    (*not* dark matter/energy, baryogenesis)
  - However, complicated; too much arbitrariness and fine-tuning:
    $\mathcal{O}(27)$ parameters (+ 2 for Majorana ν) *and* electric charges

Strong	Electromagnetic	Weak	Gravity
hadrons: p, n; pions: $\pi^{\pm}, \pi^{0}$; (QCD: quarks, gluons)	charged particles: e^{-}, μ^{-}, τ^{-}; p; $\pi^{\pm}$	p, n, π; e, μ, τ;, neutrinos: ν_e, ν_μ, ν_τ	all particles (always attractive)
nuclear binding; energy in stars	atoms, crystals, molecules; light; chemical energy	decays: $n \to pe^{-}\bar{\nu}_e$; element synthesis	weight; binding of solar system, stars, galaxies
			

Strong	Electromagnetic	Weak	Gravity
			
$V = g_\pi^2 \frac{e^{-m_\pi r}}{r}$	$\frac{e^2}{r}$	$g^2 \frac{e^{-M_W r}}{r}$	$G_N \frac{m_1 m_2}{r}$
strength: $\frac{g_\pi^2}{4\pi} \sim 14$	$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$	$\frac{g^2 E^2}{M_W^2} \sim 10^{-11}$ $(E = 1$ MeV$)$	$G_N m_1 m_2 \sim 10^{-38}$ $(m_1 = m_2 = 1$ GeV$)$
range: $\frac{\hbar}{m_\pi c} \sim 10^{-13}$ cm $\equiv$ 1 fm	∞	$\frac{\hbar}{M_W c} \sim 10^{-16}$ cm	∞
← QCD →	← Electroweak $(SU(2) \times U(1))$ →		
← Grand Unification (GUT)? →			
← Superstring? →			

- Interactions
  - Yukawa, Higgs (spin-0)
  - Gravity (spin-2)
  - Gauge (spin-1): strong, electromagnetic, weak
- Gauge (local) symmetry
  - apparently massless spin-1 (vector, gauge) bosons
  - Short range by SSB or confinement
  - Interactions ⇔ group, representations, gauge coupling
  - Unique renormalizable field theory for spin-1
  - Gauge transformations and invariant kinetic energies:

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \to \exp(i \sum_{i=1}^{N} \beta^i(x) L^i) \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \equiv e^{i\vec{\beta}\cdot\vec{L}}\psi \equiv U\psi.$$

(L^i are $n \times n$ representation matrices; β^i are real parameters)

$$[L^i, L^j] = i c_{ijk} L^k \qquad (c_{ijk} \text{ are structure constants (antisymmetric)})$$

$$\mathcal{L}_{KE_\psi} = \bar{\psi} i \not{D} \psi \equiv \bar{\psi} i \gamma^\mu \left(\partial_\mu + i g \vec{A}_\mu \cdot \vec{L} \right) \psi = \bar{\psi} i \left(\not{\partial} + i g \not{\vec{A}} \cdot \vec{L} \right) \psi$$

$$\vec{A}_\mu \cdot \vec{L} \to \vec{A}'_\mu \cdot \vec{L} \equiv U \vec{A}_\mu \cdot \vec{L} U^{-1} + \frac{i}{g} \left(\partial_\mu U \right) U^{-1} \qquad (g = \text{gauge coupling})$$

$$A'^i_\mu \sim A^i_\mu - c_{ijk} \beta^j A^k_\mu - \frac{1}{g} \partial_\mu \beta^i \qquad (\text{small } \beta^i)$$

$$\mathcal{L}_{KE_A} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} \text{ with } F^i_{\mu\nu} = \underbrace{\partial_\mu A^i_\nu - \partial_\nu A^i_\mu}_{\text{kinetic energy}} - \underbrace{g c_{ijk} A^j_\mu A^k_\nu}_{\text{self-interactions}}$$

$$\mathcal{L}_{KE_\phi} = \left(D^\mu \phi \right)^\dagger D_\mu \phi = \left(\partial^\mu \phi \right)^\dagger \partial_\mu \phi - i g A^i_\mu \left(\phi^\dagger \overleftrightarrow{\partial}^\mu L^i \phi \right) + g^2 A^{i\mu} A^j_\mu \phi^\dagger L^i L^j \phi$$

  - Explicit gauge boson mass terms $\frac{1}{2} M_A^2 \vec{A}_\mu \cdot \vec{A}^\mu$ not invariant
  - ψ, ϕ masses, scalar self-interactions, Yukawa *if* globally allowed

- Chiral fermions

$$\psi = \psi_L + \psi_R, \qquad \psi_L = P_L\psi = \frac{1-\gamma^5}{2}\psi, \qquad \psi_R = P_R\psi = \frac{1+\gamma^5}{2}\psi$$

$$P_{L,R}^2 = P_{L,R}, \qquad P_L P_R = P_R P_L = 0, \qquad P_L + P_R = I$$

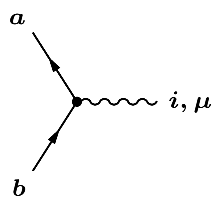
  - Chirality = helicity for relativistic ψ (corrections $\mathcal{O}(m/E)$)
  - Kinetic terms conserve chirality; mass terms flip chirality

$$\begin{aligned}\mathcal{L} &= \bar\psi\,(\,i\,\partial\!\!\!/ - m)\,\psi \\ &= \frac{1}{2}\bar\psi\, i\,\partial\!\!\!/\,(1-\gamma^5)\psi + \frac{1}{2}\bar\psi\, i\,\partial\!\!\!/\,(1+\gamma^5)\psi - \frac{m}{2}\bar\psi\,(1+\gamma^5)\psi - \frac{m}{2}\bar\psi\,(1-\gamma^5)\psi \\ &= \bar\psi_L i\,\partial\!\!\!/\,\psi_L + \bar\psi_R i\,\partial\!\!\!/\,\psi_R - m\left(\bar\psi_L\psi_R + \bar\psi_R\psi_L\right)\end{aligned}$$

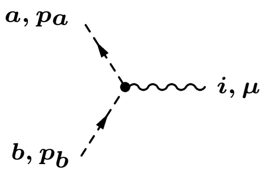
  - Chiral gauge symmetry: ψ_L and ψ_R transform under different reps, $L^i_L \neq L^i_R$

$$\mathcal{L}_{KE_\psi} = \bar\psi i D\!\!\!\!/\,\psi \equiv \bar\psi i\gamma_\mu \left(\partial^\mu + ig\vec{A}^\mu \cdot \left[\vec{L}_L P_L + \vec{L}_R P_R\right]\right)\psi$$

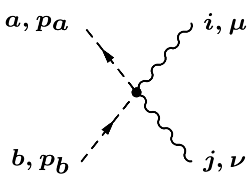
  - Fermion mass terms not allowed for chiral



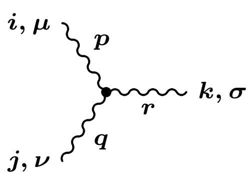
$$-ig\gamma^\mu L^i_{\psi ab} = -ig\gamma^\mu \left[L^i_{Lab}P_L + L^i_{Rab}P_R\right]$$



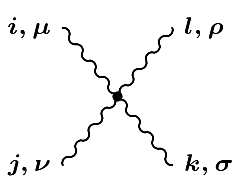
$$-ig(\bar{p}_a + \bar{p}_b)^\mu L^i_{\phi ab}$$



$$ig^2 g^{\mu\nu}\left(L^i_\phi L^j_\phi + L^j_\phi L^i_\phi\right)_{ab}$$



$$\begin{aligned} &+gc_{ijk}\left[g^{\mu\nu}(q-p)^\sigma + g^{\mu\sigma}(p-r)^\nu \right.\\ &\qquad \left. + g^{\nu\sigma}(r-q)^\mu\right] \equiv gc_{ijk}\,\mathcal{C}^{\mu\nu\sigma}(p,q,r) \end{aligned}$$



$$\begin{aligned} &-ig^2 c_{ijm}c_{klm}(g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})\\ &-ig^2 c_{ikm}c_{jlm}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\\ &-ig^2 c_{ilm}c_{jkm}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \end{aligned}$$

Figure 1: Feynman rules

- The Standard Model

  - Gauge group $SU(3) \times SU(2) \times U(1)$; gauge couplings g_s, g, g'

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

$$\begin{matrix} u_R & u_R & u_R & \nu_{eR}(?) \\ d_R & d_R & d_R & e^-_R \end{matrix}$$

  (L = left-handed, R = right-handed)

  - $SU(3)$: $u \leftrightarrow u \leftrightarrow u$, $d \leftrightarrow d \leftrightarrow d$ (8 gluons G^i)

  - $SU(2)$: $u_L \leftrightarrow d_L$, $\nu_{eL} \leftrightarrow e^-_L$ ($W^\pm$); phases (W^0)

  - $U(1)$: phases (B)

  - Heavy families: (c, s, ν_μ, μ^-), (t, b, ν_τ, τ^-)

  - $Q = T^3 + Y \rightarrow L^3_{SU(2)} + Y_{U(1)}$

- Quantum Chromodynamics (QCD)

  - Quarks: $q_{r\alpha}$, $\alpha = 1, 2, 3$ (or R,G,B); $r = u, d, s, c, b, t$ (or u_α, etc.)

$$\mathcal{L}_{SU(3)} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \sum_r \bar{q}^\alpha_r i \not{D}^\beta_\alpha q_{r\beta}$$

$$D^{\mu\beta}_\alpha = (D^\mu)_{\alpha\beta} = \partial^\mu \delta_{\alpha\beta} + i g_s\, G^{i\mu}\, L^i_{\alpha\beta}, \qquad L^i = \lambda^i/2$$

$$F^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_s f_{ijk}\, G^j_\mu\, G^k_\nu$$

  - Non-zero elements:

$$f_{123} = 1, \qquad f_{345} = \frac{1}{2}, \qquad f_{147} = \frac{1}{2}, \qquad f_{367} = -\frac{1}{2}, \qquad f_{156} = -\frac{1}{2}$$

$$f_{458} = \frac{\sqrt{3}}{2}, \qquad f_{246} = \frac{1}{2}, \qquad f_{678} = \frac{\sqrt{3}}{2}, \qquad f_{257} = \frac{1}{2}$$

$$\lambda^i = \begin{pmatrix} \tau^i & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Table 1: The $SU(3)$ (Gell-Mann) matrices. τ^i are the Pauli matrices

- The Standard Electroweak Model

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{Yuk}$$

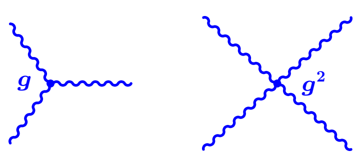
- Gauge part
$$\mathcal{L}_{gauge} = -\frac{1}{4} F^i_{\mu\nu} F^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

  - Field strength tensors:

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ F^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk} W^j_\mu W^k_\nu, \quad i = 1 \cdots 3 \end{aligned}$$

  [$g(g') = SU(2)$ $(U(1))$ gauge coupling; ϵ_{ijk} = Levi-Civita symbol]

  - Three and four-point self-interactions for the W_i:



  - B and W_3 mix to form γ, Z
  - $SU(2)$: $\Phi \to e^{i\vec{\beta}(x)\cdot\vec{L}}\Phi$; $L^i = \tau^i/2$ (doublets), $L^i = 0$ (singlets)
  - $U(1)$: $\Phi_j \to \exp(iy_j\alpha(x))\Phi_j$
    ($y_j = q_j - t^3_j$ = weak hypercharge)

- Scalar part

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger D_\mu \phi - V(\phi)$$

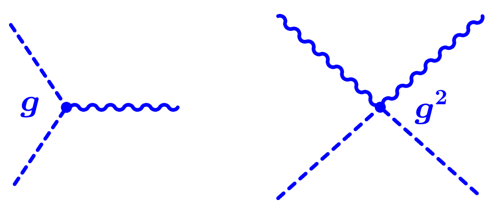
[$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is the (complex) Higgs doublet with $L^i = \tau^i/2$, $y_\phi = 1/2$]

– Gauge covariant derivative:

$$D_\mu \phi = \left(\partial_\mu + ig\frac{\tau^i}{2} W^i_\mu + \frac{ig'}{2} B_\mu \right) \phi$$

(τ^i are the Pauli matrices)

– Three and four-point gauge interactions



– Higgs potential

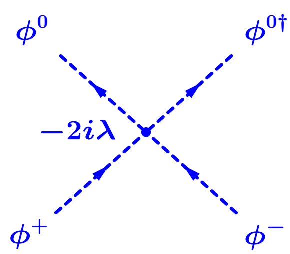
$$V(\phi) = +\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$
$$\phi^\dagger \phi = \underbrace{\phi^-}_{\phi^{+\dagger}} \phi^+ + \phi^{0\dagger} \phi^0$$

Allowed by renormalizability and gauge invariance

Spontaneous symmetry breaking for $\mu^2 < 0$

Vacuum stability: $\lambda > 0$

– Quartic self-interactions:



- Fermion part

$$\begin{aligned}
\mathcal{L}_f &= \sum_{m=1}^{3} \big(\bar{q}_{mL}\, i\, \not{D} q_{mL} + \bar{\ell}_{mL}\, i\, \not{D} \ell_{mL} \\
&\quad + \bar{u}_{mR}\, i\, \not{D} u_{mR} + \bar{d}_{mR}\, i\, \not{D} d_{mR} + \bar{e}_{mR}\, i\, \not{D} e_{mR} + \bar{\nu}_{mR}\, i\, \not{D} \nu_{mR} \big) \\
&\equiv \bar{q}_L\, i\, \not{D} q_L + \bar{\ell}_L\, i\, \not{D} \ell_L + \bar{u}_R\, i\, \not{D} u_R + \bar{d}_R\, i\, \not{D} d_R + \bar{e}_R\, i\, \not{D} e_R + \bar{\nu}_R\, i\, \not{D} \nu_R
\end{aligned}$$

– L-doublets ($L^i_L = \tau^i/2$)

$$q_{mL} = \begin{pmatrix} u_m \\ d_m \end{pmatrix}_L \qquad \ell_{mL} = \begin{pmatrix} \nu_m \\ e^-_m \end{pmatrix}_L$$

– R-singlets ($L^i_R = 0$)

$$u_{mR},\ d_{mR},\ e^-_{mR},\ \nu_{mR}(?)$$

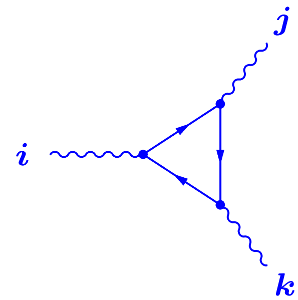
($m = 1 \cdots 3$ = family index;
-suppressed: ^{0} = weak eigenstates (definite $SU(2)$ rep.), mixtures of mass eigenstates (flavors) (e.g. u^0_{mR});
-suppressed: quark color indices (e.g., $\bar{u}^\alpha_R i \not{D} u_{\alpha R}$).)

– Can add gauge singlet ν_{mR} (for Dirac neutrino Yukawa)

– Different (chiral) L and R representations lead to parity and charge conjugation violation (maximal for $SU(2)$)

– Fermion mass terms forbidden by chiral symmetry

– Anomalies absent for chosen hypercharges and 3 colors
(includes quark-lepton cancellations)

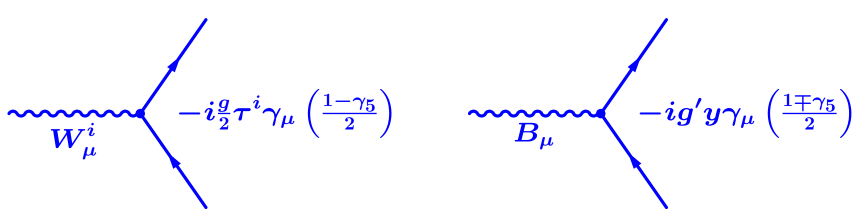


$$
\begin{aligned}
A_{ijk} &= 2\mathrm{Tr}\, L_L^i\{L_L^j, L_L^k\} - 2\mathrm{Tr}\, L_R^i\{L_R^j, L_R^k\} = 0 \\
T_i &= \mathrm{Tr}\, L_L^i - \mathrm{Tr}\, L_R^i = 0
\end{aligned}
$$

– Gauge covariant derivatives ($y = q - t_L^3$)

$$
\begin{aligned}
D_\mu q_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W_\mu^i + i\frac{g'}{6}B_\mu\right) q_{mL} & \left[q = \pm\frac{1}{2} + \frac{1}{6} = \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}\right] \\
D_\mu \ell_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W_\mu^i - i\frac{g'}{2}B_\mu\right) \ell_{mL} & \left[q = \pm\frac{1}{2} - \frac{1}{2} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right] \\
D_\mu u_{mR} &= \left(\partial_\mu + i\frac{2}{3}g' B_\mu\right) u_{mR} & \left[q = 0 + \frac{2}{3} = \frac{2}{3}\right] \\
D_\mu d_{mR} &= \left(\partial_\mu - i\frac{g'}{3}B_\mu\right) d_{mR} & \left[q = 0 - \frac{1}{3} = -\frac{1}{3}\right] \\
D_\mu e_{mR} &= \left(\partial_\mu - ig' B_\mu\right) e_{mR} & [q = 0 - 1 = -1] \\
D_\mu \nu_{mR} &= \left(\partial_\mu\right) \nu_{mR} & [q = 0 + 0 = 0]
\end{aligned}
$$

– Read off W and B couplings to fermions:

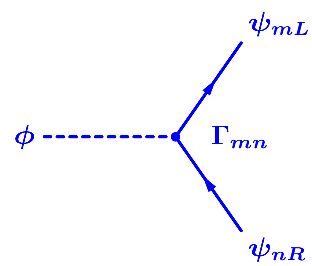


- Yukawa couplings (couple L to R) (restore weak eigenstate label ^{0})

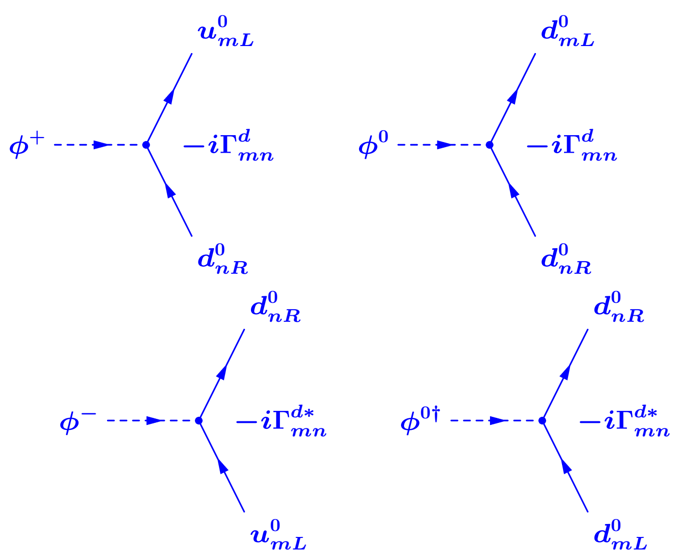
$$-\mathcal{L}_{Yuk} = \sum_{m,n=1}^{3} \Big[\Gamma^u_{mn} \bar{q}^0_{mL} \tilde{\phi} u^0_{nR} + \Gamma^d_{mn} \bar{q}^0_{mL} \phi d^0_{nR} + \Gamma^e_{mn} \bar{\ell}^0_{mL} \phi e^0_{nR} + \Gamma^\nu_{mn} \bar{\ell}^0_{mL} \tilde{\phi} \nu^0_{nR} \Big] + h.c.$$

$$\equiv \bar{q}^0_L \Gamma^u \tilde{\phi} u^0_R + \bar{q}^0_L \Gamma^d \phi d^0_R + \bar{\ell}^0_L \Gamma^l \phi e^0_R + \bar{\ell}^0_L \Gamma^\nu \tilde{\phi} \nu^0_R + h.c.$$

- Γ_{mn} are completely arbitrary Yukawa matrices, which determine fermion masses and mixings
- d, e terms require doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ with $y_\phi = 1/2$



$$\Gamma^d_{mn} \bar{q}^0_{mL} \phi d^0_{nR} = \Gamma^d_{mn} \left[\bar{u}^0_{mL} \phi^+ d^0_{nR} + \bar{d}^0_{mL} \phi^0 d^0_{nR} \right]$$



– u (and ν) terms require doublet $\Phi = \begin{pmatrix} \Phi^0 \\ \Phi^- \end{pmatrix}$ with $y_\Phi = -1/2$
(otherwise Y and Q violated)

$$-\mathcal{L}_{Yuk} = \bar{q}_L^0 \Gamma^u \Phi u_R^0 + \bar{q}_L^0 \Gamma^d \phi d_R^0 + \bar{\ell}_L^0 \Gamma^l \phi e_R^0 + \bar{\ell}_L^0 \Gamma^\nu \Phi \nu_R^0 + h.c.$$

– In $SU(2)$ the 2 and 2^* are similar $\Rightarrow \tilde{\phi} \equiv i\tau^2 \phi^\dagger = \begin{pmatrix} \phi^{0\dagger} \\ -\phi^- \end{pmatrix}$ transforms as a 2 with $y_{\tilde{\phi}} = -\frac{1}{2}$ ($\Rightarrow$ only one doublet needed)

– Does *not* generalize to $SU(3)$, most extra $U(1)'$, supersymmetry, $SO(10)$ etc ($\Rightarrow$ need two doublets)
(does generalize to $SU(2)_L \times SU(2)_R \times U(1)$ and $SU(5)$)